Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence
Online Appendix

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1 Final Good Producers

Final good producers use the following CES aggregator to produce.

$$Y_t^i = \left(\int_0^{A_t} (Y_{mt}^j)^{\frac{1}{\vartheta}} dj \right)^{\vartheta} \tag{1.1}$$

Let p_{mt} be the real price of intermediate goods. Cost minimization determines the following real marginal cost,

$$MC_t = \frac{p_{mt}}{A_t^{\vartheta - 1}}$$

Let P_t^i be the nominal price of final good i and P_t the nominal price level. The demand curve facing each final good producer is:

$$Y_t^i = \left(\frac{P_t^i}{P_t} \right)^{-\mu_t/(\mu_t - 1)} Y_t \tag{1.2}$$

where the price index is given by:

$$P_t = \left(\int_0^1 (P_t^i)^{-1/(\mu_t - 1)} di \right)^{-(\mu_t - 1)}, \tag{1.3}$$

We assume Calvo pricing with indexing. Let $1 - \xi_p$ be the i.i.d probability that a firm is able to re-optimize its price and let $\pi_t = P_t / P_{t-1}$ be the inflation rate. Firms that are unable to re-optimize during the period adjust their price according to the following indexing rule:

$$P_t^i = P_{t-1}^i \pi_{t-1}^{\iota_p} \pi^{1 - \iota_p} \tag{1.4}$$

where π is the steady state inflation rate and ι_p reflects the degree of indexing to lagged inflation.

For firms able to re-optimize, the optimization problem is to choose a new reset price P_t^* to maximize expected discounted profits until the next re-optimization, given by

$$E_t \sum_{\tau=0}^{\infty} \xi_p^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_t^* \Gamma_{t,t+\tau}}{P_{t+\tau}} - MC_{t+\tau} \right) Y_{t+\tau}^i \tag{1.5}$$

subject to the demand function (1.2) and where

$$\Gamma_{t,t+\tau} \equiv \prod_{k=1}^{\tau} \pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p} \tag{1.6}$$

The first order condition for P_t^* is

$$0 = E_t \sum_{\tau=0}^{\infty} \xi_p^{\tau} \Lambda_{t,t+\tau} \left[\frac{P_t^* \Gamma_{t,t+\tau}}{P_{t+\tau}} - \mu_{t+\tau} MC_{t+\tau} \right] Y_{t+\tau}^i$$

replacing $Y_{t+\tau}^i = \left[\frac{P_t^* \Gamma_{t,t+\tau}}{P_{t+\tau}} \right]^{\frac{-\mu_t}{\mu_t - 1}} Y_{t+\tau}$,

$$0 = E_t \sum_{\tau=0}^{\infty} \xi_p^{\tau} \Lambda_{t,t+\tau} \left[\frac{P_t^* \Gamma_{t,t+\tau}}{P_{t+\tau}} - \mu_{t+\tau} MC_{t+\tau} \right] \left[\frac{P_t^* \Gamma_{t,t+\tau}}{P_{t+\tau}} \right]^{\frac{-\mu_t}{\mu_t - 1}} Y_{t+\tau} \tag{1.7}$$

The price index that relates P_t to P_t^*, P_{t-1} and π_{t-1} is then:

$$P_t = \left[(1 - \xi_p) \left(P_t^* \right)^{-1/(\mu_t - 1)} + \xi_p \left(\pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{t-1} \right)^{-1/(\mu_t - 1)} \right]^{-(\mu_t - 1)} \tag{1.8}$$

Equations (1.7) and (1.8) jointly determine inflation. In the loglinear equilibrium, current inflation is a function of current real marginal cost MC_t, expected future inflation, and lagged inflation.

2 Employment Agencies and Wage Adjustment

The household is a monopolistically competitive supplier of labor. Think of the household as supplying its labor to form a labor composite. Firms then hire the labor composite. The only difference from the standard DSGE model with wage rigidity, is that households now supply two types of labor, skilled and unskilled.

Let $X_t = \{L_t, L_{st}\}$ denote a labor composite. As is standard, we assume that X_t is the following CES aggregate of the differentiated types of labor that households provide

$$X_t = \left[\int_0^1 X_t^{h\,\frac{1}{\mu_{wt}}} dh \right]^{\mu_{wt}}. \tag{2.1}$$

where $\mu_{wt} > 1$ obeys an exogenous stochastic process[1].

[1] In estimating the model we introduce wage markup shocks to the wage setting problem of unskilled labor only, so the markup for skilled labor is constant at its steady state level.

Let W_{xt} denote the wage of the labor composite and let W_{xt}^h be the nominal wage for labor supplied by household h. Then profit maximization by competitive employment agencies yields the following demand for type x labor:

$$X_t^h = \left(\frac{W_{xt}^h}{W_{xt}}\right)^{-\mu_{wt}/(\mu_{wt}-1)} X_t, \tag{2.2}$$

with

$$W_{xt} = \left[\int_0^1 W_{xt}^{h\,-\frac{1}{\mu_{wt}-1}} dh\right]^{-(\mu_{wt}-1)}. \tag{2.3}$$

As with price setting by final goods firms, we assume that households engage in Calvo wage setting with indexation. Each period a fraction $1-\xi_w$ of households re-optimize their wage. Households who are not able to re-optimize adjust according to the following indexing rule:

$$W_{xt} = W_{xt-1}\pi_{t-1}^{\iota_w}\pi^{1-\iota_w}(1+\gamma_y) \tag{2.4}$$

where $(1+\gamma_y)$ is the steady state growth rate of labor productivity.

The remaining fraction of households choose an optimal reset wage W_t^* by maximizing

$$E_t\left\{\sum_{\tau=0}^{\infty}\xi_w^\tau\beta^\tau\left[-\upsilon_x\frac{\left(X_{t+\tau}^h\right)^{1+\varphi}}{1+\varphi}+u'(C_{t+\tau})\frac{W_{xt}^*\Gamma_{wt,t+\tau}}{P_{t+\tau}}X_{t+\tau}^h\right]\right\} \tag{2.5}$$

subject to the demand for type h labor and where $\upsilon_x = \{\upsilon, \upsilon_s\}$ and the indexing factor $\Gamma_{wt,t+\tau}$ is given by

$$\Gamma_{wt,t+\tau} \equiv \prod_{k=1}^{\tau}\pi_{t+k-1}^{\iota_w}\pi^{1-\iota_w}(1+\gamma_y) \tag{2.6}$$

The first order condition for the re-set wage and the equation for the composite wage index as a function of the reset wage, inflation and the lagged wage are given, respectively, by

$$E_t\left\{\sum_{\tau=0}^{\infty}\xi_w^\tau\Lambda_{t,\tau}\left[\frac{W_{xt}^*\Gamma_{wt,t+\tau}}{P_{t+\tau}}-\mu_{wt}\upsilon\left(\frac{W_{xt}^*\Gamma_{wt,t+\tau}}{W_{xt+\tau}}\right)^{\frac{-\varphi\mu_{wt}}{\mu_{wt}-1}}\frac{X_{t+\tau}^{\varphi}}{u'(C_{t+\tau})}\right]\left(\frac{W_{xt}^*\Gamma_{wt,t+\tau}}{W_{xt+\tau}}\right)^{\frac{-\mu_{wt}}{\mu_{wt}-1}}X_{t+\tau}\right\}=0 \tag{2.7}$$

$$W_{xt} = \left[(1-\xi_w)\left(W_{xt}^*\right)^{-1/(\mu_{wt}-1)}+\xi_p\left((1+\gamma_y)\pi_{t-1}^{\iota_w}\pi^{1-\iota_w}W_{xt-1}\right)^{-1/(\mu_{wt}-1)}\right]^{-(\mu_{wt}-1)} \tag{2.8}$$

3 Aggregation

We aggregate equations related to individual intermediate good producer using the CES aggregator in (1.1) and the fact that up to a first order approximation $Y_t^i = Y_t$. Define aggregate labor demand, aggregate capital , average capital utilization rate as,

$$L_t = A_t L_t^j$$
$$K_t = A_t K_t^j$$
$$U_t = U_t^j$$

In the last two expressions we are using the fact that all intermediate producers are symmetric. Given this symmetry, note the following relationship between intermediate output and aggregate final output.

$$\begin{aligned} Y_t &= \left(\int_0^{A_t} (Y_{mt})^{\frac{1}{\vartheta}} dj \right)^{\vartheta} \\ &= A_t^{\vartheta} Y_{mt}^j \\ &= A_t^{\vartheta-1} \theta_t (U_t K_t)^{\alpha} (L_t)^{1-\alpha} \end{aligned}$$

where the last step uses the intrmediate goods production function $Y_{mt}^j = \theta_t (U_t^j K_t^j)^{\alpha} (L_t^j)^{1-\alpha}$.

Given the information above we can express the factor demands from intermediate good producers in aggregate terms.

$$\alpha \frac{p_{mt} Y_{mt}^j}{K_t^j} = \varsigma [D_t + \delta(U_t^j) Q_t]$$

$$\alpha \frac{p_{mt} Y_{mt}^j}{U_t^j} = \varsigma \delta'(U_t^j) Q_t K_t^j$$

$$(1-\alpha) \frac{p_{mt} Y_{mt}^j}{L_t^j} = \varsigma w_t$$

Applying the CES aggregator to the FOCs we get,

$$\alpha \frac{p_{mt} A_t^{\vartheta} Y_{mt}^j}{K_t} = \varsigma A_t^{\vartheta} [D_t + \delta(U_t^j) Q_t]$$

$$\alpha \frac{p_{mt} A_t^{\vartheta} Y_{mt}^j}{U_t^j} = \varsigma A_t^{\vartheta} \delta'(U_t^j) Q_t K_t^j$$

$$(1-\alpha) \frac{p_{mt} A_t^{\vartheta} Y_{mt}^j}{L_t^j} = A_t^{\vartheta} \varsigma w_t$$

Replacing to get the FOCs in terms of aggregate variables and real marginal cost of final producers we get

$$\alpha \frac{MC_t Y_t}{K_t} = \varsigma [D_t + \delta(U_t) Q_t]$$

$$\alpha \frac{MC_t Y_t}{U_t} = \varsigma \delta'(U) Q_t K_t$$

$$(1-\alpha) \frac{MC_t Y_t}{L_t} = \varsigma w_t$$

The value of an adopted technology can also be expressed in terms of aggregate output and final good producer real marginal cost. That value is defined as

$$\begin{aligned} V_t &= \Pi_{mt} + \phi E_t \{\Lambda_{t,t+1} V_{t+1}\} \\ &= (\varsigma - 1) \frac{p_{mt}}{\varsigma} Y_{mt} + \phi E_t \{\Lambda_{t,t+1} V_{t+1}\} \\ &= \left(\frac{\varsigma - 1}{\varsigma} \right) MC_t A_t^{\vartheta - 1} Y_{mt} + \phi E_t \{\Lambda_{t,t+1} V_{t+1}\} \end{aligned}$$

Multiplying both terms by A_t we get,

$$V_t A_t = \left(\frac{\varsigma - 1}{\varsigma} \right) MC_t Y_t + \phi E_t \{\Lambda_{t,t+1} V_{t+1} A_{t+1} \frac{A_t}{A_{t+1}}\}$$

Renaming $V_t A_t = V_t^A$

$$V_t^A = \left(\frac{\varsigma - 1}{\varsigma} \right) MC_t Y_t + \phi E_t \{\Lambda_{t,t+1} V_{t+1}^A \frac{A_t}{A_{t+1}}\}$$

We can modify the value of unadopted technologies to incorporate V_t^A. Scaling J_t by Z_t

$$J_t Z_t = -w_{st} L_{sat} Z_t + \phi E_t \{\Lambda_{t,t+1} [\lambda_t V_{t+1} A_{t+1} \frac{Z_t}{A_{t+1}} + (1-\lambda_t) J_{t+1} Z_{t+1} \frac{Z_t}{Z_{t+1}}]\}$$

Defining $J_t Z_t \equiv J_t^Z$ and $L_{sat}^Z \equiv L_{sat} Z_t$

$$J_t^Z = -w_{st} L_{sat}^Z + \phi E_t \left\{ \Lambda_{t,t+1} \left[\lambda_t V_{t+1}^A \frac{Z_t}{A_{t+1}} + (1-\lambda_t) J_{t+1}^Z \frac{Z_t}{Z_{t+1}} \right] \right\}$$

We also modify the $R\&D$ and adoption FOCs accordingly.

$$E_t \left\{ \Lambda_{t,t+1} J_{t+1}^Z \chi_t \frac{Z_t}{Z_{t+1}} L_{srt}^{\rho_z - 1} \right\} = w_{st}$$

$$\rho_\lambda \lambda_t \phi E_t \left\{ \Lambda_{t,t+1} \left[V_{t+1}^A \frac{Z_t}{A_{t+1}} - J_{t+1}^Z \frac{Z_t}{Z_{t+1}} \right] \right\} = w_{st} L_{sat}^Z$$

4 Model Equations

The basic model consists of 35 equations related to 35 variables (27 endogenous and 8 shock processes). The variables are described in the following table.

Variable	Description
MC_t	Real marginal cost final good prod
D_t	Ex depreciation dividends
Q_t	Price of capital
U_t	captial utilization
K_t	Captial stock
Y_t	Real Aggregate Output
w_t	Real wage unskilled labor
w_{st}	Real wage skilled labor
L_t	Unskilled Labor
A_t	Stock of adopted technologies
C_t	Consumption
R_t	Real risk free interest rate
$\Lambda_{t,t+1}$	Stoch Discount Factor
I_t	Investment
R_{nt}	Nominal Interest Rate
π_t	Gross inflation rate
Z_t	Stock of technologies
L_{srt}	Skilled labor demand: R&D sector

L_{st}	Skilled labor supply
L_{sat}^Z	Skilled labor demand: adoption sector, scaled by Z_t ($L_{sat}Z_t$)
J_t^Z	Value unadopted good scaled by Z_t (J_tZ_t)
V_t^A	Value adopted good scaled by A_t (V_tA_t)
λ_t	Adoption probability
p_t^*	optimal relative price (P_t^*/P_{t-1})
w_t^*	optimal real wage (W_t^*/P_t) unskilled labor
w_{st}^*	optimal real wage (W_{st}^*/P_t) skilled labor
u_{ct}	Marginal utility consumption
G_t	Government Spending
θ_t	Productivity Shock
μ_t	Mark up shock
μ_{wt}	Wage mark up shock
p_{kt}	Price of capital shock
r_t^m	Taylor rule shock
χ_t	R&D productivity shock
ζ_t	Liquidity demand shock

The model equations are the following,

$$\alpha\frac{MC_tY_t}{K_t} = \varsigma[D_t + \delta(U_t)Q_t] \tag{4.1}$$

$$\alpha\frac{MC_tY_t}{U_t} = \varsigma\delta'(U_t)Q_tK_t \tag{4.2}$$

$$(1-\alpha)\frac{MC_tY_t}{L_t} = \varsigma w_t \tag{4.3}$$

$$Y_t = \left[A_t^{\vartheta-1}\theta_t\right](U_tK_t)^\alpha(L_t)^{1-\alpha} \tag{4.4}$$

$$u_{ct} = \frac{1}{C_t - bC_{t-1}} - \beta\mathbb{E}_t\frac{b}{C_{t+1} - bC_t} \tag{4.5}$$

$$1 = E_t\{\Lambda_{t,t+1}R_t\} + \zeta_t \tag{4.6}$$

$$1 = E_t\left\{\Lambda_{t,t+1}\frac{D_{t+1} + Q_{t+1}}{Q_t}\right\} \tag{4.7}$$

$$\Lambda_{t,t+1} = \beta E_t\frac{u_{ct+1}}{u_{ct}} \tag{4.8}$$

$$\frac{Q_t}{p_{kt}} = 1 + f\left(\frac{I_t}{(1+\gamma_y)I_{t-1}}\right) + \frac{I_t}{(1+\gamma_y)I_{t-1}} f'\left(\frac{I_t}{(1+\gamma_y)I_{t-1}}\right)$$

$$- E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{(1+\gamma_y)I_t}\right)^2 f'\left(\frac{I_{t+1}}{(1+\gamma_y)I_t}\right) \tag{4.9}$$

$$K_{t+1} = I_t + (1 - \delta(U_t))K_t \tag{4.10}$$

$$R_{nt} = r_t^m \left(\left(\frac{\pi_t}{\pi^0}\right)^{\phi_\pi} \left(\frac{L_t}{L^{ss}}\right)^{\phi_y} R_n\right)^{1-\rho^R} (R_{nt-1})^{\rho^R} \tag{4.11}$$

$$R_{nt} = R_t E_t \pi_{t+1} \tag{4.12}$$

$$Z_{t+1} = \chi_t Z_t L_{srt}^{\rho_z} + \phi Z_t \tag{4.13}$$

$$E_t \left\{ \Lambda_{t,t+1} J_{t+1}^Z \chi_t \frac{Z_t}{Z_{t+1}} L_{srt}^{\rho_z - 1} \right\} = w_{st} \tag{4.14}$$

$$J_t^Z = -w_{st} L_{sat}^Z + \phi E_t \left\{ \Lambda_{t,t+1} \left[\lambda_t V_{t+1}^A \frac{Z_t}{A_{t+1}} + (1-\lambda_t) J_{t+1}^Z \frac{Z_t}{Z_{t+1}} \right] \right\} \tag{4.15}$$

$$\lambda_t = \kappa_\lambda (L_{sat}^Z)^{\rho_\lambda} \tag{4.16}$$

$$A_{t+1} = \lambda_t \phi [Z_t - A_t] + \phi A_t \tag{4.17}$$

$$V_t^A = \left(\frac{\varsigma - 1}{\varsigma}\right) MC_t Y_t + \phi E_t \{ \Lambda_{t,t+1} V_{t+1}^A \frac{A_t}{A_{t+1}} \} \tag{4.18}$$

$$\rho_\lambda \lambda_t \phi E_t \left\{ \Lambda_{t,t+1} \left[V_{t+1}^A \frac{Z_t}{A_{t+1}} - J_{t+1}^Z \frac{Z_t}{Z_{t+1}} \right] \right\} = w_{st} L_{sat}^Z \tag{4.19}$$

$$0 = E_t \sum_{\tau=0}^{\infty} \xi_p^\tau \Lambda_{t,t+\tau} \left[\frac{p_t^* \Gamma_{t,t+\tau}}{\prod_{k=0}^{\tau} \pi_{t+k}} - \mu_{t+\tau} MC_{t+\tau} \right] \left[\frac{p_t^* \Gamma_{t,t+\tau}}{\prod_{k=0}^{\tau} \pi_{t+k}} \right]^{\frac{-\mu_t}{\mu_t - 1}} Y_{t+\tau} \tag{4.20}$$

$$\pi_t = \left[(1-\xi_p) (p_t^*)^{-1/(\mu_t - 1)} + \xi_p \left(\pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^{-1/(\mu_t - 1)} \right]^{-(\mu_t - 1)} \tag{4.21}$$

$$E_t \left\{ \sum_{\tau=0}^{\infty} \xi_w^\tau \Lambda_{t,\tau} \left[\frac{w_t^* \Gamma_{wt,t+\tau}}{\prod_{k=1}^{\tau} \pi_{t+k}} - \mu_{wt} \upsilon \left(\frac{w_t^* \Gamma_{wt,t+\tau}}{w_{t+\tau} \prod_{k=1}^{\tau} \pi_{t+k}} \right)^{\frac{-\varphi \mu_{wt}}{\mu_{wt} - 1}} \frac{L_{t+\tau}^\varphi}{u'(C_{t+\tau})} \right] \left(\frac{w_t^* \Gamma_{t,t+\tau}}{w_{t+\tau} \prod_{k=1}^{\tau} \pi_{t+k}} \right)^{\frac{-\mu_{wt}}{\mu_{wt} - 1}} L_{t+\tau} \right\} = 0 \tag{4.22}$$

$$w_t = \left[(1-\xi_w) (w_t^*)^{-1/(\mu_{wt} - 1)} + \xi_p \left((1+\gamma_y) \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \frac{w_{t-1}}{\pi_t} \right)^{-1/(\mu_{wt} - 1)} \right]^{-(\mu_{wt} - 1)} \tag{4.23}$$

$$E_t \left\{ \sum_{\tau=0}^{\infty} \xi_w^\tau \Lambda_{t,\tau} \left[\frac{w_{st}^* \Gamma_{wt,t+\tau}}{\prod_{k=1}^{\tau} \pi_{t+k}} - \mu_w \upsilon_s \left(\frac{w_{st}^* \Gamma_{wt,t+\tau}}{w_{st+\tau} \prod_{k=1}^{\tau} \pi_{t+k}} \right)^{\frac{-\varphi \mu_w}{\mu_w - 1}} \frac{L_{st+\tau}^\varphi}{u'(C_{t+\tau})} \right] \left(\frac{w_{st}^* \Gamma_{t,t+\tau}}{w_{st+\tau} \prod_{k=1}^{\tau} \pi_{t+k}} \right)^{\frac{-\mu_w}{\mu_w - 1}} L_{st+\tau} \right\} = 0 \tag{4.24}$$

$$w_{st} = \left[(1-\xi_w)\left(w_{st}^*\right)^{-1/(\mu_w-1)} + \xi_p \left((1+\gamma_y)\pi_{t-1}^{\iota_w}\pi^{1-\iota_w}\frac{w_{st-1}}{\pi_t} \right)^{-1/(\mu_w-1)} \right]^{-(\mu_w-1)} \tag{4.25}$$

$$Y_t = C_t + p_{kt}\left[1 + f\left(\frac{I_t}{(1+\gamma_y)I_{t-1}}\right)\right] I_t + G_t \tag{4.26}$$

$$L_{st} = \left[1 - \frac{A_t}{Z_t}\right] L_{sat}^Z + L_{srt} \tag{4.27}$$

$$\log(G_t/(1+\gamma_y)^t) = (1-\rho_g)\bar{g} + \rho_g \log(G_{t-1}/(1+\gamma_y)^{t-1}) + \sigma_g \epsilon_t^g \tag{4.28}$$

$$\log(\theta_t) = \rho_\theta \log(\theta_{t-1}) + \sigma_\theta \epsilon_t^\theta \tag{4.29}$$

$$\log(\mu_t) = (1-\rho_\mu)\mu + \rho_\mu \log(\mu_{t-1}) + \sigma_\mu \epsilon_t^\mu \tag{4.30}$$

$$\log(\mu_{wt}) = (1-\rho_{\mu_w})\mu_w + \rho_{\mu_w} \log(\mu_{wt-1}) + \sigma_{\mu_w} \epsilon_t^{\mu_w} \tag{4.31}$$

$$\log(p_{kt}) = \rho_{p_k} \log(p_{kt-1}) + \sigma_{p_k} \epsilon_t^{p_k} \tag{4.32}$$

$$\log(r_t^m) = \rho_{r^m} \log(r_{t-1}^m) + \sigma_{r^m} \epsilon_t^{r^m} \tag{4.33}$$

$$\log(\chi_t) = (1-\rho_\chi)\log(\bar{\chi}) + \rho_\chi \log(\chi_{t-1}) + \sigma_\chi \epsilon_t^\chi \tag{4.34}$$

$$\zeta_t = (1-\rho_\zeta)\bar{\zeta} + \rho_\zeta \zeta_{t-1} + \sigma_\zeta \epsilon_t^\zeta \tag{4.35}$$

5 Stationarizing the Model

The model presented in section 4 is non-stationary. In fact, it is straightforward to see that if there is positive skilled labor in equilibrium A_t and Z_t grow at a rate $1+\gamma_a$ in steady state. Moreover, Y_t, C_t, K_t, I_t, G_t, real wages and values V_t^A and J_t^Z grow at a rate $1+\gamma_y \equiv (1+\gamma_a)^{\frac{\vartheta-1}{1-\alpha}}$ in steady state.

Hence, we stationarize variables dividing by either $(1+\gamma_y)^t$ or $(1+\gamma_a)^t$ depending on the case. In particular, define

$$\tilde{X}_t \equiv \frac{X_t}{(1+\gamma_j)^t}$$

for $j = \{y, a\}$ depending on the steady state growth rate of the variable. Applying the normalization to the model described in 4 we get the following

$$\alpha \frac{MC_t \tilde{Y}_t}{\tilde{K}_t} = \varsigma[D_t + \delta(U_t)Q_t] \tag{5.1}$$

$$\alpha \frac{MC_t \tilde{Y}_t}{U_t} = \varsigma \delta'(U_t) Q_t \tilde{K}_t \tag{5.2}$$

$$(1-\alpha) \frac{MC_t \tilde{Y}_t}{L_t} = \varsigma \tilde{w}_t \tag{5.3}$$

$$\tilde{Y}_t = \tilde{A}_t^{\vartheta - 1} \theta_t (U_t \tilde{K}_t)^{\alpha} (L_t)^{1-\alpha} \tag{5.4}$$

$$\tilde{u}_{ct} = (1+\gamma_y)^t u_{ct} = \frac{(1+\gamma_y)}{(1+\gamma_y)\tilde{C}_t - b\tilde{C}_{t-1}} - \beta \mathbb{E}_t \frac{b}{(1+\gamma_y)\tilde{C}_{t+1} - b\tilde{C}_t} \tag{5.5}$$

$$1 = E_t \left\{ \frac{\tilde{\Lambda}_{t,t+1}}{1+\gamma_y} R_t \right\} + \zeta_t \tag{5.6}$$

$$1 = E_t \left\{ \frac{\tilde{\Lambda}_{t,t+1}}{1+\gamma_y} \frac{D_{t+1} + Q_{t+1}}{Q_{t-1}} \right\} \tag{5.7}$$

$$\tilde{\Lambda}_{t,t+1} = \beta E_t \frac{\tilde{u}_{ct+1}}{\tilde{u}_{ct}} \Rightarrow \Lambda_{t,t+1} = \tilde{\Lambda}_{t,t+1} \frac{1}{1+\gamma_y} \tag{5.8}$$

$$\frac{Q_t}{p_{kt}} = 1 + f\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right) + \frac{\tilde{I}_t}{\tilde{I}_{t-1}} f'\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right) - E_t \Lambda_{t,t+1} \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right)^2 f'\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right) \tag{5.9}$$

$$(1+\gamma_y)\tilde{K}_{t+1} = \tilde{I}_t + (1 - \delta(U_t))\tilde{K}_t \tag{5.10}$$

$$R_{nt} = r_t^m \left(\left(\frac{\pi_t}{\pi^0}\right)^{\phi_\pi} \left(\frac{L_t}{L^{ss}}\right)^{\phi_y} R_n \right)^{1-\rho^R} (R_{nt-1})^{\rho^R} \tag{5.11}$$

$$R_{nt} = R_t E_t \pi_{t+1} \tag{5.12}$$

$$(1+\gamma_a)\tilde{Z}_{t+1} = \chi_t \tilde{Z}_t L_{srt}^{\rho_z} + \phi \tilde{Z}_t \tag{5.13}$$

$$E_t \left\{ \tilde{\Lambda}_{t,t+1} \tilde{J}_{t+1}^Z \chi_t \frac{\tilde{Z}_t}{\tilde{Z}_{t+1}(1+\gamma_a)} L_{srt}^{\rho_z - 1} \right\} = \tilde{w}_{st} \tag{5.14}$$

$$\tilde{J}_t^Z = -\tilde{w}_{st} L_{sat}^Z + \phi E_t \left\{ \frac{\tilde{\Lambda}_{t,t+1}}{1+\gamma_a} \left[\lambda_t \tilde{V}_{t+1}^A \frac{\tilde{Z}_t}{\tilde{A}_{t+1}} + (1-\lambda_t) \tilde{J}_{t+1}^Z \frac{\tilde{Z}_t}{\tilde{Z}_{t+1}} \right] \right\} \tag{5.15}$$

$$\lambda_t = \kappa_\lambda (L_{sat}^Z)^{\rho_\lambda} \tag{5.16}$$

$$(1+\gamma_a)\tilde{A}_{t+1} = \lambda_t \phi [\tilde{Z}_t - \tilde{A}_t] + \phi \tilde{A}_t \tag{5.17}$$

$$\tilde{V}_t^A = \left(\frac{\varsigma - 1}{\varsigma}\right) MC_t \tilde{Y}_t + \phi E_t \{ \tilde{\Lambda}_{t,t+1} \tilde{V}_{t+1}^A \frac{\tilde{A}_t}{\tilde{A}_{t+1}(1+\gamma_a)} \} \tag{5.18}$$

$$\rho_\lambda \lambda_t \phi E_t \left\{ \frac{\tilde{\Lambda}_{t,t+1}}{1+\gamma_a} \left[\tilde{V}^A_{t+1} \frac{\tilde{Z}_t}{\tilde{A}_{t+1}} - \tilde{J}^Z_{t+1} \frac{\tilde{Z}_t}{\tilde{Z}_{t+1}} \right] \right\} = \tilde{w}_{st} L^Z_{sat} \tag{5.19}$$

$$0 = E_t \sum_{\tau=0}^{\infty} \xi_p^\tau \tilde{\Lambda}_{t,t+\tau} \left[\frac{p_t^* \Gamma_{t,t+\tau}}{\prod_{k=0}^{\tau} \pi_{t+k}} - \mu_{t+\tau} MC_{t+\tau} \right] \left[\frac{p_t^* \Gamma_{t,t+\tau}}{\prod_{k=0}^{\tau} \pi_{t+k}} \right]^{\frac{-\mu_t}{\mu_t - 1}} \tilde{Y}_{t+\tau} \tag{5.20}$$

$$\pi_t = \left[(1-\xi_p) \left(p_t^* \right)^{-1/(\mu_t - 1)} + \xi_p \left(\pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^{-1/(\mu_t - 1)} \right]^{-(\mu_t - 1)} \tag{5.21}$$

$$E_t \sum_{\tau=0}^{\infty} \xi_w^\tau \tilde{\Lambda}_{t,t+\tau} \left[\frac{\tilde{w}_t^* \Gamma_{wt,t+\tau}}{\prod_{k=1}^{\tau} \pi_{t+k} (1+\gamma_y)^\tau} - \mu_{wt} \upsilon \left(\frac{\tilde{w}_t^* \Gamma_{wt,t+\tau}}{\tilde{w}_{t+\tau} \prod_{k=1}^{\tau} \pi_{t+k} (1+\gamma_y)^\tau} \right)^{\frac{-\varphi \mu_{wt}}{\mu_{wt} - 1}} \frac{L^\varphi_{t+\tau}}{\tilde{u}_{ct+\tau}} \right]$$
$$\left(\frac{\tilde{w}_t^* \Gamma_{t,t+\tau}}{\tilde{w}_{t+\tau} \prod_{k=1}^{\tau} \pi_{t+k} (1+\gamma_y)^\tau} \right)^{\frac{-\mu_{wt}}{\mu_{wt} - 1}} L_{t+\tau} = 0 \tag{5.22}$$

$$\tilde{w}_t = \left[(1-\xi_w) \left(\tilde{w}_t^* \right)^{-1/(\mu_{wt} - 1)} + \xi_p \left(\frac{\pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \tilde{w}_{t-1}}{\pi_t} \right)^{-1/(\mu_{wt} - 1)} \right]^{-(\mu_{wt} - 1)} \tag{5.23}$$

$$E_t \sum_{\tau=0}^{\infty} \xi_w^\tau \tilde{\Lambda}_{t,t+\tau} \left[\frac{\tilde{w}_{st}^* \Gamma_{wt,t+\tau}}{\prod_{k=1}^{\tau} \pi_{t+k} (1+\gamma_y)^\tau} - \mu_w \upsilon_s \left(\frac{\tilde{w}_{st}^* \Gamma_{wt,t+\tau}}{\tilde{w}_{st+\tau} \prod_{k=1}^{\tau} \pi_{t+k} (1+\gamma_y)^\tau} \right)^{\frac{-\varphi \mu_w}{\mu_w - 1}} \frac{L^\varphi_{st+\tau}}{\tilde{u}_{ct+\tau}} \right]$$
$$\left(\frac{\tilde{w}_{st}^* \Gamma_{t,t+\tau}}{\tilde{w}_{st+\tau} \prod_{k=1}^{\tau} \pi_{t+k} (1+\gamma_y)^\tau} \right)^{\frac{-\mu_w}{\mu_w - 1}} L_{st+\tau} = 0 \tag{5.24}$$

$$\tilde{w}_{st} = \left[(1-\xi_w) \left(\tilde{w}_{st}^* \right)^{-1/(\mu_w - 1)} + \xi_p \left(\frac{\pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \tilde{w}_{st-1}}{\pi_t} \right)^{-1/(\mu_w - 1)} \right]^{-(\mu_w - 1)} \tag{5.25}$$

$$\tilde{Y}_t = \tilde{C}_t + p_{kt} \left[1 + f \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \right) \right] \tilde{I}_t + \tilde{G}_t \tag{5.26}$$

$$L_{st} = \left[1 - \frac{\tilde{A}_t}{\tilde{Z}_t} \right] L^Z_{sat} + L_{srt} \tag{5.27}$$

$$\log(\tilde{G}_t) = (1-\rho_g)\bar{g} + \rho_g \log(\tilde{G}_{t-1}) + \sigma_g \epsilon^g_t \tag{5.28}$$

$$\log(\theta_t) = \rho_\theta \log(\theta_{t-1}) + \sigma_\theta \epsilon^\theta_t \tag{5.29}$$

$$\log(\mu_t) = (1-\rho_\mu)\mu + \rho_\mu \log(\mu_{t-1}) + \sigma_\mu \epsilon^\mu_t \tag{5.30}$$

$$\log(\mu_{wt}) = (1 - \rho_{\mu_w})\mu_w + \rho_{\mu_w} \log(\mu_{wt-1}) + \sigma_{\mu_w} \epsilon_t^{\mu_w} \tag{5.31}$$

$$\log(p_{kt}) = \rho_{p_k} \log(p_{kt-1}) + \sigma_{p_k} \epsilon_t^{p_k} \tag{5.32}$$

$$\log(r_t^m) = \rho_{r^m} \log(r_{t-1}^m) + \sigma_{r^m} \epsilon_t^{r^m} \tag{5.33}$$

$$\log(\chi_t) = (1 - \rho_\chi) \log(\bar{\chi}) + \rho_\chi \log(\chi_{t-1}) + \sigma_\chi \epsilon_t^\chi \tag{5.34}$$

$$\zeta_t = (1 - \rho_\zeta)\bar{\zeta} + \rho_\zeta \zeta_{t-1} + \sigma_\zeta \epsilon_t^\zeta \tag{5.35}$$

6 Steady State

We log-linearize the equilibrium around a steady state where output grows at a rate $(1+\gamma_a)^{\frac{\vartheta-1}{1-\alpha}}$, that is,

$$1 + \gamma_y = (1 + \gamma_a)^{\frac{\vartheta-1}{1-\alpha}}$$

where $1+\gamma_a$ is the gross growth rate of A_t in steady state. Assuming, capital depreciation is given by,

$$\delta(U) = \delta - \frac{d_1}{1+\omega} + d_1 \frac{U^{1+\omega}}{1+\omega}$$

Where δ is the steady state depreciation rate and d_1 is calibrated such that $U = 1$ in steady state. For a given steady state output growth rate $1 + \gamma_y$ and inflation rate π we compute the steady state in the following steps.

From the definition $1 + \gamma_y$

$$1 + \gamma_a = (1 + \gamma_y)^{\frac{1-\alpha}{\vartheta-1}}$$

From (5.6), (5.8), (5.9) and (5.7)

$$R = \frac{(1 + \gamma_y)(1 - \bar{\zeta})}{\beta}$$

$$\tilde{\Lambda} = \beta$$

$$Q = 1$$

$$D = R - 1$$

Setting $U = 1$, from (5.1) and (5.2)

$$d_1 = R - 1 + \delta$$

From (5.20), (5.21) and (5.12)

$$MC = 1/\mu$$

$$p^* = \pi$$

$$R_n = R\pi$$

From (5.1), (5.3) and (5.4)

$$\frac{\tilde{K}}{\tilde{Y}} = \frac{\alpha}{\mu\varsigma(R - 1 + \delta)}$$

$$\frac{\tilde{w}L}{\tilde{Y}} = \frac{1 - \alpha}{\mu\varsigma}$$

From (5.10)

$$\frac{\tilde{I}}{\tilde{Y}} = (\delta + \gamma_y)\frac{\tilde{K}}{\tilde{Y}}$$

For a given government consumption/ GDP ratio G/Y and using (5.26)

$$\frac{\tilde{C}}{\tilde{Y}} = 1 - \frac{\tilde{I}}{\tilde{Y}} - \frac{G}{Y}$$

From (5.5)

$$\tilde{Y}\tilde{u}_c = \frac{1}{\tilde{C}/\tilde{Y}} \frac{1 + \gamma_y - \beta b}{1 + \gamma_y - b}$$

From (5.22), by setting $L = 1$ we can calibrate the unskilled labor disutility parameter υ

$$\upsilon = \frac{\tilde{w}L}{\tilde{Y}} \left(\tilde{Y}\tilde{u}_c\right) \frac{1}{\mu_w}$$

Now from (5.4), normalizing $\tilde{A} = 1$ in steady state and using $L = 1$

$$\tilde{Y} = \left(\frac{\tilde{K}}{\tilde{Y}}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha}{\mu\varsigma(R - 1 + \delta)}\right)^{\frac{\alpha}{1-\alpha}}$$

Given the calculations up to this point , we are also able to compute $\tilde{K}$, $\tilde{w}$, $\tilde{I}$, $\tilde{C}$ and $\tilde{u}_c$.

Now from (5.18)

$$\tilde{V}^A = \left(\frac{1+\gamma_a}{1+\gamma_a-\phi\beta}\right)\left(\frac{\varsigma-1}{\varsigma\mu}\right)\tilde{Y}$$

Given a value for steady state adoption probability λ and that $\tilde{A}=1$, (5.17) implies

$$\tilde{Z} = \frac{1+\gamma_a-\phi}{\lambda\phi}+1$$

From (5.15) and (5.19)

$$\tilde{J}^Z = \frac{(1-\rho_\lambda)\lambda\phi\beta}{1+\gamma_a-(1-\lambda+\rho_\lambda\lambda)\phi\beta}\tilde{V}^A\tilde{Z}$$

And going back to (5.19) we get,

$$\tilde{w}_s L_{sa}^Z = \frac{\rho_\lambda\lambda\phi\beta}{1+\gamma_a}\left[\tilde{V}^A\tilde{Z}-\tilde{J}^Z\right]$$

From (5.13) and (5.14)

$$\tilde{w}_s L_{sr} = \beta\frac{\tilde{J}^Z(1+\gamma_a-\phi)}{1+\gamma_a}$$

Using (5.27)

$$\tilde{w}_s L_s = \left[1-\frac{1}{\tilde{Z}}\right]\tilde{w}_s L_{sa}^Z + \tilde{w}_s L_{sr}$$

Now going to (5.24), we can calibrate υ_s to set $L_s = 1$

$$\upsilon_s = \frac{(\tilde{w}_s L_s)\tilde{u}_c}{\mu_w}$$

Given that $L_s = 1$, we can get from the last expressions $\tilde{w}_s$, L_{sr} and L_{sa}^Z. Lastly, we calibrate the values for R&D productivity χ and κ_λ to make the growth rates γ_a (or γ_y) and adoption probability λ in steady state consistent with the model. From (5.16) and (5.13)

$$\kappa_\lambda = \frac{\lambda}{(L_{sa}^Z)^{\rho_\lambda}}$$

$$\chi = \frac{1+\gamma_a-\phi}{L_{sr}^{\rho_z}}$$

Note that from equation (5.23) and (5.25) in steady state $\tilde{w} = \tilde{w}^*$ and $\tilde{w}_s = \tilde{w}_s^*$

7 Log-linearized model

We loglinearize the stationary model and estimate parameters using Dynare. The loglinearized equations are shown below. Lower case and hatted variables are log-deviations from the non-stochastic steady state.

$$\alpha\beta\left[\hat{mc}_t + \hat{y}_t - \hat{k}_t\right] = \frac{\varsigma\mu\tilde{K}}{\tilde{Y}}\left(1+\gamma_y-\beta\right)\hat{d}_t + \alpha\beta\hat{u}_t + \frac{\beta\varsigma\mu\tilde{K}}{\tilde{Y}}\delta\hat{q}_t \tag{7.1}$$

$$\hat{mc}_t + \hat{y}_t = (1+\omega)\hat{u}_t + \hat{q}_t + \hat{k}_t \tag{7.2}$$

$$\hat{mc}_t + \hat{y}_t - \hat{l}_t = \hat{w}_t \tag{7.3}$$

$$\hat{y}_t = (\vartheta - 1)\hat{a}_t + \hat{\theta}_t + \alpha\hat{k}_t + \alpha\hat{u}_t + (1-\alpha)\hat{l}_t \tag{7.4}$$

$$\begin{aligned}(1+\gamma_y-b)(1+\gamma_y-\beta b)\widehat{u}_{ct} = &-\left[(1+\gamma_y)^2+\beta b^2\right]\hat{c}_t - (1+\gamma_y)b\hat{c}_{t-1}\\ &+\beta(1+\gamma_y)b\hat{c}_{t+1}\end{aligned} \tag{7.5}$$

$$0 = \hat{\Lambda}_{t,t+1} + \hat{r}_t + \hat{\zeta}_t \tag{7.6}$$

$$0 = (1+\gamma_y)\widehat{\Lambda}_{t,t+1} - (1+\gamma_y)\hat{q}_t + (1+\gamma_y-\beta)\hat{d}_{t+1} + \beta\hat{q}_{t+1} \tag{7.7}$$

$$\hat{\Lambda}_{t,t+1} = \hat{u}_{ct+1} - \hat{u}_{ct} \tag{7.8}$$

$$\hat{q}_t - \hat{p}_{kt} = \left(1 + \frac{\beta}{1+\gamma_y}\right)\psi\hat{i}_t - \psi\hat{i}_{t-1} - \frac{\psi\beta}{1+\gamma_y}\hat{i}_{t+1} \tag{7.9}$$

$$\beta(1+\gamma_y)\frac{\tilde{K}}{\tilde{Y}}\hat{k}_{t+1} = \beta\frac{\tilde{I}}{\tilde{Y}}\hat{i}_t + \beta(1-\delta)\frac{\tilde{K}}{\tilde{Y}}\hat{k}_t - \left(1+\gamma_y-\beta(1-\delta)\right)\frac{\tilde{K}}{\tilde{Y}}\hat{u}_t \tag{7.10}$$

$$\hat{r}_{nt} = \left(1-\rho^R\right)\left[\phi_\pi\hat{\pi}_t + \phi_y\hat{l}_t\right] + \rho^R\hat{r}_{nt-1} + \hat{r}^m_t \tag{7.11}$$

$$\hat{r}_{nt} = \hat{r}_t + \hat{\pi}_{t+1} \tag{7.12}$$

$$(1+\gamma_a)\hat{z}_{t+1} = (1+\gamma_a-\phi)\left(\hat{\chi}_t + \rho_z\hat{l}_{srt}\right) + (1+\gamma_a)\hat{z}_t \tag{7.13}$$

$$\hat{\Lambda}_{t,t+1} + \hat{j}^Z_{t+1} + \hat{\chi}_t + \hat{z}_t - \hat{z}_{t+1} + (\rho_z - 1)\hat{l}_{srt} = \hat{w}_{st} \tag{7.14}$$

$$\begin{aligned}\tilde{J}^Z\hat{j}^Z_t = &-\tilde{w}_s L^Z_{sa}\left(\hat{w}_{st} + \hat{l}^Z_{sat}\right) + \frac{\phi\beta}{1+\gamma_a}\left[\lambda\tilde{V}^A\tilde{Z} + (1-\lambda)\tilde{J}^Z\right]\hat{\Lambda}_{tt+1} - \frac{\phi\beta}{1+\gamma_a}(1-\lambda)\tilde{J}^Z\hat{z}_{t+1}\\ &+\frac{\phi\beta}{1+\gamma_a}\left[\tilde{V}^A\tilde{Z} - \tilde{J}^Z\right]\lambda\hat{\lambda}_t + \frac{\phi\beta}{1+\gamma_a}\lambda\tilde{V}^A\tilde{Z}\left(\hat{v}^A_{t+1} - \hat{a}_{t+1}\right)\\ &+\frac{\phi\beta}{1+\gamma_a}\left[\lambda\tilde{V}^A\tilde{Z} + (1-\lambda)\tilde{J}^Z\right]\hat{z}_t + \frac{\phi\beta}{1+\gamma_a}(1-\lambda)\tilde{J}^Z\hat{j}^Z_{t+1}\end{aligned} \tag{7.15}$$

$$\hat{\lambda}_t = \rho_\lambda \hat{l}^Z_{sat} \tag{7.16}$$

$$(1+\gamma_a)\,\hat{a}_{t+1} = (1+\gamma_a-\phi)\,\hat{\lambda}_t + \lambda\phi\tilde{Z}\hat{z}_t - (\lambda-1)\,\phi\hat{a}_t \tag{7.17}$$

$$\frac{\tilde{V}^A}{\tilde{Y}}\hat{v}^A_t = \left(\frac{\varsigma-1}{\varsigma\mu}\right)(\hat{mc}_t+\hat{y}_t) + \frac{\phi\beta}{1+\gamma_a}\frac{\tilde{V}^A}{\tilde{Y}}\left(\hat{\Lambda}_{tt+1}+\hat{v}^A_{t+1}+\hat{a}_t-\hat{a}_{t+1}\right) \tag{7.18}$$

$$\begin{aligned}\tilde{w}_s L^Z_{sa}\left(\hat{w}_{st}+\hat{l}^Z_{sat}-\hat{\lambda}_t-\hat{\Lambda}_{tt+1}\right) &= \rho_\lambda\lambda\frac{\phi\beta}{1+\gamma_a}\tilde{V}^A Z\left(\hat{v}^A_{t+1}+\hat{z}_t-\hat{a}_{t+1}\right)\\ &\quad-\rho_\lambda\lambda\frac{\phi\beta}{1+\gamma_a}\tilde{J}^Z\left(\hat{j}^Z_{t+1}+\hat{z}_t-\hat{z}_{t+1}\right)\end{aligned} \tag{7.19}$$

From (5.20) and (5.21) we get the price Phillips Curve,

$$\hat{\pi}_t = \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p(1+\beta\iota_p)}(\hat{\mu}_t+\hat{mc}_t) + \frac{\beta}{1+\beta\iota_p}\hat{\pi}_{t+1} \tag{7.20}$$

From (5.22) and (5.23) we get the wage Phillips Curve,

$$\begin{aligned}(1+\kappa_w)\,\hat{w}_t &= \frac{1}{1+\beta}\left(\hat{w}_{t-1}+\iota_w\hat{\pi}_{t-1}-(1+\beta\iota_w)\,\hat{\pi}_t\right)+\left(\varphi\hat{l}_t-\hat{u}_{ct}\right)\kappa_w\\ &\quad+\frac{\beta}{1+\beta}\left(\hat{\pi}_{t+1}+\hat{w}_{t+1}\right)+\kappa_w\hat{\mu}_{wt}\end{aligned} \tag{7.21}$$

where

$$\kappa_w \equiv \frac{(1-\xi_w\beta)(1-\xi_w)}{\left(\frac{\varphi}{1-1/\mu_w}+1\right)\xi_w(1+\beta)}$$

From (5.24) and (5.25) we get the skilled labor wage Phillips Curve,

$$\begin{aligned}(1+\kappa_w)\,\hat{w}_{st} &= \frac{1}{1+\beta}\left(\hat{w}_{st-1}+\iota_w\hat{\pi}_{t-1}-(1+\beta\iota_w)\,\hat{\pi}_t\right)+\left(\varphi\hat{l}_{st}-\hat{u}_{ct}\right)\kappa_w\\ &\quad+\frac{\beta}{1+\beta}\left(\hat{\pi}_{t+1}+\hat{w}_{st+1}\right)\end{aligned} \tag{7.22}$$

Goods and Skilled labor market clearing conditions,

$$\hat{y}_t = \frac{\tilde{C}}{\tilde{Y}}\hat{c}_t + \frac{\tilde{I}}{\tilde{Y}}\left(\hat{p}_{kt}+\hat{i}_t\right) + \frac{\tilde{G}}{\tilde{Y}}\hat{g}_t \tag{7.23}$$

$$\tilde{Z}L_s\hat{l}_{st} = L^Z_{sa}(\hat{z}_t-\hat{a}_t) + \left[\tilde{Z}-1\right]L^Z_{sa}\hat{l}^Z_{sat} + \tilde{Z}L_{sr}\hat{l}_{srt} \tag{7.24}$$

Shocks processes,

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \sigma_g\epsilon^g_t \tag{7.25}$$

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \sigma_\theta \epsilon_t^\theta \tag{7.26}$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \sigma_\mu \epsilon_t^\mu \tag{7.27}$$

$$\hat{\mu}_{wt} = \rho_{\mu_w} \hat{\mu}_{wt-1} + \sigma_{\mu_w} \epsilon_t^{\mu_w} \tag{7.28}$$

$$\hat{p}_{kt} = \rho_{p_k} \hat{p}_{kt-1} + \sigma_{p_k} \epsilon_t^{p_k} \tag{7.29}$$

$$\hat{r}_t^m = \rho_{r^m} \hat{r}_{t-1}^m + \sigma_{r^m} \epsilon_t^{r^m} \tag{7.30}$$

$$\hat{\chi}_t = \rho_\chi \hat{\chi}_{t-1} + \sigma_\chi \epsilon_t^\chi \tag{7.31}$$

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \sigma_\zeta \epsilon_t^\zeta \tag{7.32}$$

8 ρ_λ Robustness Check

Figure 1 plots the evolution of A_t for the baseline value of ρ_λ (0.925) and a lower value (0.85) (see Section 4.7 in the paper for a discussion of the effect of varying ρ_λ).

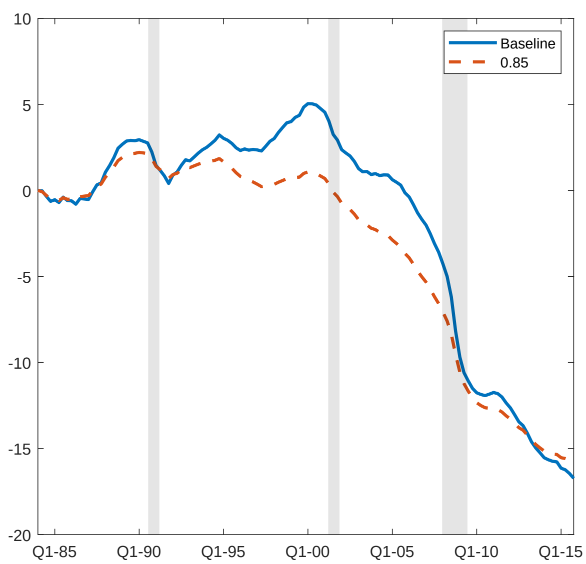


Figure 1: Estimated time series for stock of adopted technologies A_t for baseline and alternative calibrations of ρ_λ

9 More IRFs

Figure 2: Impulse Response to 1 std. dev. Money Shock

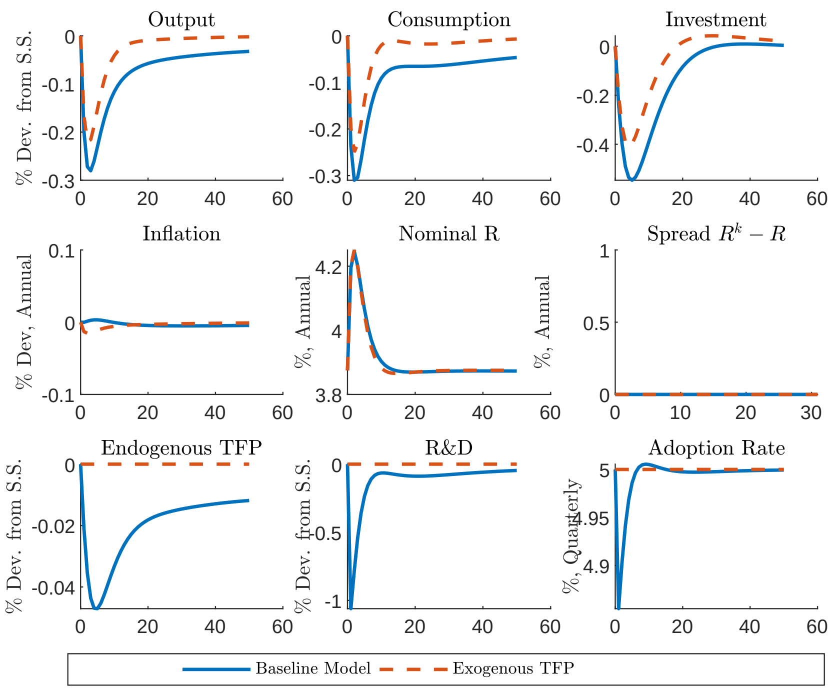
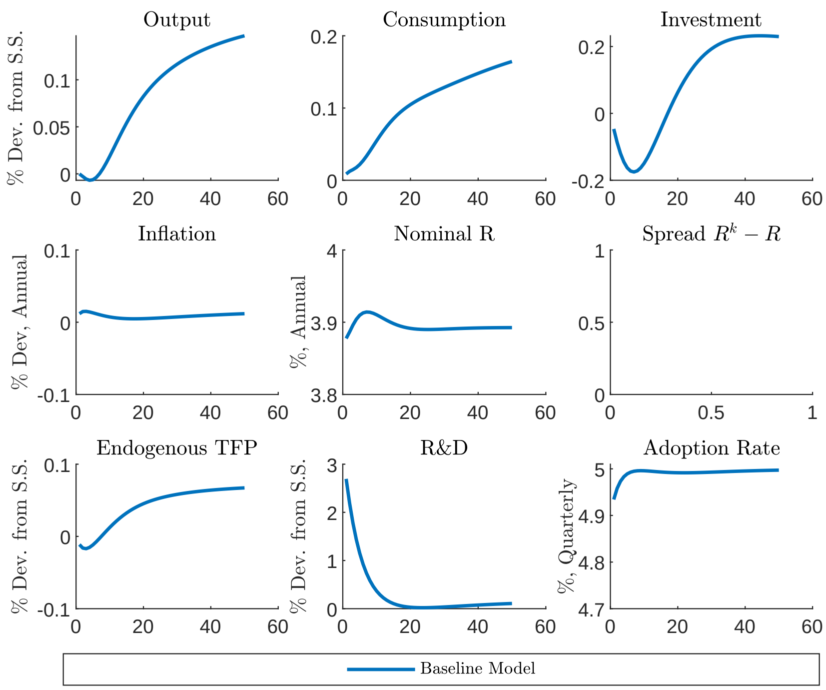


Figure 3: Impulse Response to 1 std. dev. R&D Productivity Shock



10 Additional Tables

As a check of the fit of the estimated model, Table 2 presents the theoretical standard deviations of the observable variables generated by the model and compares them with the data in our sample. Roughly speaking the model is in line with the actual volatilities of the key variables.

Table 2: Comparison of Standard Deviations

Variable	Data	Model
Output Growth	0.55	0.63
Consumption Growth	0.51	0.71
Investment Growth	1.54	1.52
Inflation	0.23	0.36
Nominal R	0.60	0.55
Hours (level)	1.82	1.53
R&D Expenditure Growth	4.00	6.83

Table 3: Prior and Posterior Distributions of Shock Processes

Parameter	Description	Prior			Posterior	
		Distr	Mean	St. Dev.	Mean	St. Dev.
σ_ζ	Liq. Demand	Inv. Gamma	0.10	2.00	0.225	0.0013
σ_χ	R&D	Inv. Gamma	0.10	2.00	2.202	0.1749
σ_g	Govt. exp.	Inv. Gamma	0.10	2.00	2.559	0.0350
σ_{mp}	Monetary	Inv. Gamma	0.10	2.00	0.097	0.0001
σ_μ	Markup	Inv. Gamma	0.10	2.00	0.093	0.0002
σ_{pk}	Investment	Inv. Gamma	0.10	2.00	1.270	0.0092
σ_θ	TFP	Inv. Gamma	0.10	2.00	0.489	0.0013
σ_{μ_w}	Wage markup	Inv. Gamma	0.10	2.00	0.284	0.0014
ρ_ζ	Liq. Demand	Beta	0.50	0.20	0.924	0.0006
ρ_χ	R&D	Beta	0.50	0.20	0.803	0.0073
ρ_g	Govt. exp.	Beta	0.50	0.20	0.968	0.0001
ρ_{mp}	Monetary	Beta	0.50	0.20	0.465	0.0068
ρ_μ	Markup	Beta	0.50	0.20	0.401	0.0186
ρ_{pk}	Investment	Beta	0.50	0.20	0.899	0.0014
ρ_θ	TFP	Beta	0.50	0.20	0.953	0.0007
ρ_{μ_w}	Wage markup	Beta	0.50	0.20	0.288	0.0101